Lesson 2-5: Solving Equations with the Variable on Each Side

1 Variables on Each Side

To solve an equation that has variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side.

Example 1 Solve an Equation with Variables on Each Side

Solve $2 + 5k = 3k - 6$. Check your solution.

$2 + 5k = 3\cancel{k} - 6$	Original equation
$-3k = -\cancel{3k}$	subtraction prop.
$\cancel{2} + 2k = -6$	simplify
$-\cancel{2} \quad = -2$	subtraction prop.
$2k = -8$	simplify
$\frac{\cancel{2}k}{\cancel{2}} = \frac{-8}{2}$	division prop.
$k = -4$	simplify

CHECK $2 + 5k = 3k - 6$	Original equation
$2 + 5(-4) \stackrel{?}{=} 3(-4) - 6$	substitution
$2 + -20 \stackrel{?}{=} -12 - 6$	simplify
$-18 = -18$ ✓	simplify

EXERCISE 1

Solve each equation. Check your solution.

1A. $5a + 2 = 6 - \cancel{7a}$

$+7a \qquad +\cancel{7a}$

$12a + \cancel{2} = 6$

$-\cancel{2} \quad -2$

$\frac{\cancel{12}a}{\cancel{12}} = \frac{4}{12}$

$a = 0.33$

$a = \frac{1}{3}$

1B. $1.3c = 3.3c + 2.8$

$-1.3c \quad -1.3c$

$0 = 2c + \cancel{2.8}$

$-2.8 \qquad -\cancel{2.8}$

$\frac{-2.8}{2} = \frac{\cancel{2}c}{\cancel{2}}$

$\rightarrow -1.4 = c$

1C. $\frac{x}{2} + 1 = \frac{1}{4}x - 6 \rightarrow -14 + 1 = -7 - 6$

$-13 = -13$

$\frac{x}{2} = \frac{1}{2}x = 0.5x$

$\frac{1}{4}x = 0.25x$

$0.5x + 1 = 0.\cancel{25}x - 6$

$-0.25x \quad -0.\cancel{25}x$

$0.25x + \cancel{1} = -6$

$-\cancel{1} \quad -1$

$\frac{0.\cancel{25}x}{0.\cancel{25}} = \frac{-7}{0.25}$

$x = -28$

2 Grouping Symbols If equations contain grouping symbols such as parentheses or brackets, use the Distributive Property first to remove the grouping symbols.

Example 2 Solve an Equation with Grouping Symbols

Solve $6(5m - 3) = \frac{1}{3}(24m + 12)$.

$6(5m - 3) = \frac{1}{3}(24m + 12)$	Original equation
$30m - 18 = 8m + 4$	distributive prop
$30m - 18 - 8m = \cancel{8m} + 4 - \cancel{8m}$	Subtraction prop.
$22m - 18 = 4$	simplify
$22m - \cancel{18} + \cancel{18} = 4 + 18$	addition prop.
$22m = 22$	simplify
$\frac{\cancel{22}m}{\cancel{22}} = \frac{22}{22}$	division prop.
$m = 1$	simplify

EXERCISE 2

Solve each equation. Check your solution.

2A. $8s - 10 = 3(6 - 2s)$

$8s - 10 = 18 - \cancel{6s}$
$+6s \quad\quad +\cancel{6s}$

$14s - \cancel{10} = 18$
$+\cancel{10} \quad +10$

$\frac{\cancel{14}s}{\cancel{14}} = \frac{28}{14}$

$s = 2$

2B. $7(n - 1) = -2(3 + n)$

$7n - 7 = -6 - \cancel{2n}$
$+2n \quad\quad +\cancel{2n}$

$9n - \cancel{7} = -6$
$+\cancel{7} \quad +7$

$\frac{\cancel{9}n}{\cancel{9}} = \frac{1}{9}$

$n = \frac{1}{9}$

Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Some equations are true for all values of the variables. These are called **identities**.

Example 3 Find Special Solutions

Solve each equation.

a. $5x + 5 = 3(5x - 4) - 10x$

$5x + 5 = 3(5x - 4) - 10x$	Original equation
$5x + 5 = 15x - 12 - 10x$	distributive prop.
$\cancel{5x} + 5 = \cancel{5x} - 12$	combined like terms
$-\cancel{5x} \quad = -\cancel{5x}$	subtraction prop.
$5 \neq -12$	

left side $\neq$ right side

Since $5 \neq -12$, this equation has no solution.

$\varnothing$

b. $3(2b - 1) - 7 = 6b - 10$

$3(2b - 1) - 7 = 6b - 10$	Original equation
$6b - 3 - 7 = 6b - 10$	distributive prop.
$\cancel{6b} - 10 = \cancel{6b} - 10$	Simplify
$-\cancel{6b} \quad -\cancel{6b}$	subtraction and addition prop.
$0 = 0$	

$-10 = -10$
$+10 \quad +10$

left side = right side

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of b.

EXERCISE 3

Solve each equation.

3A. $7x + 5(x - 1) = -5 + 12x$

$7x + 5x - 5 = -5 + 12x$

$\cancel{12x} - 5 = -5 + \cancel{12x}$

$-\cancel{12x} \quad\quad -\cancel{12x}$

$-5 = -5$

identity

3B. $6(y - 5) = 2(10 + 3y)$

$\cancel{6y} - 30 = 20 + \cancel{6y}$

$-\cancel{6y} \quad\quad -\cancel{6y}$

$-30 \neq 20$

no solution

$\varnothing$

The steps for solving an equation can be summarized as follows.

ConceptSummary Steps for Solving Equations

Step 1 Simplify the expressions on each side. Use the Distributive Property as needed.

Step 2 Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.

Step 3 Use the Multiplication or Division Property of Equality to solve.